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Modelling slope failure using a quasi-static MPM with a non-local strain softening approach

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Abstract

The Material Point Method (MPM), which can be thought of as a mesh-free technique, has been shown to be very efficient in avoiding the mesh distortion problem in large deformation analyses. However, for the widely used explicit dynamic MPM formulation, the time step must inevitably be very small in order to guarantee convergence, especially in the case of quasi-static problems. In this paper, an incremental updated Lagrangian quasi-static MPM formulation is developed, which requires less computation effort by using much larger time steps. Issues pertaining to the implementation of the present MPM formulation are discussed. Strain softening, which may potentially lead to localisation phenomena, is also considered in the constitutive model. Scale effects and mesh size dependency in the solution are accounted for by applying a spatial averaging approach to the strains using a weighting function defined by an internal length scale characterising the non-local deformation. A progressive failure of a slope is simulated in order to demonstrate the efficiency and good performance of the proposed formulation.

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Keywords: MPM; quasi-static; implicit; non-local; strain softening; slope stability.

1. Introduction

Several strategies, including the Arbitrary Lagrangian-Eulerian (ALE) method [1] and mesh-free techniques [2], have been adopted to avoid the mesh distortion problem in large deformation analyses. A method called Particle in Cell, originally developed in the field of computational fluid mechanics, has also been modified and applied to solid mechanics by Sulsky et al. [3] which is now known as the Material Point Method (MPM). This method can be categorised as a mesh-free technique; however, since it consists of both Lagrangian and convective phases, it can also be considered an ALE method. A further difficulty arising in many geotechnical applications is the strain softening response of the soil. Under plastic deformation, the mechanical strength of dense soils reduces with plastic strain until a residual value is reached. Numerical simulation of soils demonstrating this phenomena has been shown to suffer from mesh dependency. In fact, it has been observed that the finer the mesh is, the softer the material response will be [4]. Several attempts have also been made to account for strain softening in numerical simulations and different algorithms were proposed to eliminate the effect of mesh dependency [5–7]. In this study, a quasi-

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static MPM formulation was developed for two-dimensional 4-node linear and 8-node quadratic elements based on an incrementally updated Lagrangian formulation (UL) with Jaumann stress rate [8]. The strain softening of the constitutive model is considered by reduction of the strength parameters of the soil with the increase of plastic strain. In order to account for scale effects and to eliminate any mesh size dependency, the Mohr-Coulomb yield function was used within the framework of a non-local elasto-viscoplastic constitutive model. The accuracy of the formulation was evaluated by simulating the progressive failure of a slope in order to demonstrate the efficiency and good performance of the proposed formulation.

2. Quasi-static MPM Formulation

The material point method requires two levels of discretisation for Lagrangian and Eulerian phases. The body is discretised into a set of Lagrangian material points (MPs) which carry the permanent data including position, mass, density, volume, stresses and state variables. An Eulerian background mesh, which can be generated using any conventional FEM element type, is also adopted for calculation purposes (but no data is assigned to this mesh permanently). Fig. 1 shows how the particles, in a Lagrangian formulation, move through the Eulerian mesh.

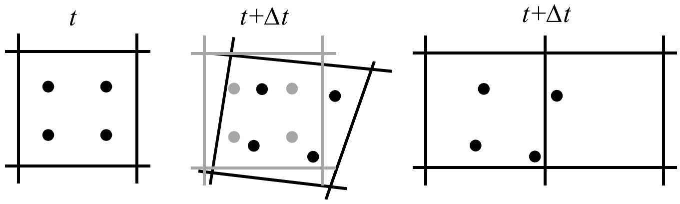

Fig. 1. Description of a continuum using MPM.

2.1. Updated Lagrangian formulation

The equilibrium equation for the updated Lagrangian formulation can be written the following matrix form:

$$\int_{V_t} \underbrace{\boldsymbol{B}_L^T \boldsymbol{C}_{ep} \boldsymbol{B}_L dV_t}_{\text{small strain stiffness matrix}} + \underbrace{\int_{V_t} \boldsymbol{B}_L^T \bar{\bar{\boldsymbol{\sigma}}}_t \bar{\boldsymbol{B}}_L dV_t + \int_{V_t} \boldsymbol{B}_{NL}^T \bar{\boldsymbol{\sigma}}_t \boldsymbol{B}_{NL} dV_t}_{\text{geometrical stiffness matrix}} = \underbrace{\boldsymbol{R}_{t+\Delta t}}_{\text{external force vector}} - \underbrace{\int_{V_t} \boldsymbol{B}_L^T \hat{\boldsymbol{\sigma}}_t dV_t}_{\text{internal force vector}} \quad (1)$$

The first term in the left hand side is the conventional small strain stiffness matrix, the second and third terms are the geometrical stiffness matrix, the first term in right hand side is the total external force including body force and surface traction, and the integration in the right hand side represents the internal force vector which can contain the existing stress reaction and plastic correction. For more details on the different matrices in Eq. 1 and the corresponding implicit iterative solution procedure, the reader is referred for example to [1,9].

2.2. Mapping, re-mapping and the numerical integration

In order to mitigate the well-known problem of particle crossing noise the mixed integration method is adopted in which Gauss integration, for fully filled elements, and material point integration for the partially filled elements around the boundary are considered [10]. All the elements inside the body are assumed to be fully filled. An element at the boundary is assessed and if the total MP volume in that element is less than 80% of the element volume, it is considered partially filled.

To adopt the mixed integration approach a mapping/re-mapping procedure should be designed to exchange the state variables between MPs and the Gauss points (GPs) in fully filled elements. For the case of linear elements with one GP, this step is very straightforward. A weighted average of all the MPs inside an element can be assigned to the GP and the updated values obtained on the GP can be considered for all the MPs inside that element. For 8-node quadratic elements with four GPs, a polynomial function with four unknowns can be determined to represent any variable distribution inside an element. An analytical solution can be achieved to obtain the interpolation functions because the number and position of the integration points, which contain the known values of the desired variable, are fixed within an element. Considering the order of the element, the authors of this work suggested the following polynomial for stresses at any point with the local coordinates of (ζ,η) inside an 8-node quadratic element:

$$\sigma_{ij}(\zeta,\eta) = \alpha^0_{ij} + \alpha^1_{ij}\zeta + \alpha^2_{ij}\eta + \alpha^3_{ij}\zeta\eta \tag{2}$$

The stress state at any point within an element can also be written as:

$$\sigma_{ij}(\zeta,\eta) = H_k\sigma^k_{ij} \tag{3}$$

where H_k are the interpolation functions, σ^k_{ij} is the stress state at GPs, and $k = 1, \ldots, n$ where n is the total number of GPs inside the element.

For fully filled elements, the MPs data are also required to be transferred to the GPs. As the position and the number of MPs are not fixed inside an element the previous approach can not be implemented. A least square averaging can be applied to find the best fitted function for each variable over an element. To keep the consistency of mapping and re-mapping, the same polynomial (Eq. 2) is considered.

2.3. Particle splitting algorithm

Another issue regarding mixed integration is the ill-conditioning problem. This issue can occur for both linear and higher order elements. A linear element with one GP or MP can produce ill-conditioning if there are not enough number of constraints on the element's degrees of freedom. For higher order elements the same problem can be observed when the number of MPs inside a partially filled element is not enough or the MPs are positioned on one straight line. In order to mitigate this problem, a particle splitting algorithm can be applied. Here, when required, a MP will be divided into four smaller voxels to generate enough integration points. The state variables of the parent MP are assigned to the new MPs. To avoid excessive increase in the number of MPs around the boundary, at the end of the load step the newly generated MPs are merged and form the parent MP with the average values of their new state variables. The geometry of a particle can be traced using the displacement of the center of the particle voxel and the deformation gradient. The positions of four new split particles can be obtained as follow:

$$\begin{pmatrix} x^t \\ y^t \end{pmatrix} = \begin{pmatrix} x^0 \\ y^0 \end{pmatrix} + \begin{pmatrix} u^t_x \\ u^t_y \end{pmatrix} + \boldsymbol{F}^t \begin{pmatrix} \pm L_x/4 \\ \pm L_y/4 \end{pmatrix} \tag{4}$$

where superscript 0 and t represent the initial and current time, the vectors (x, y) and (u_x, u_y) are the position and the total displacement of the particle, $\boldsymbol{F}^t$ is the deformation gradient and L_x and L_y are the initial dimensions of the particle voxel.

3. Non-local plasticity

Implementation of plasticity algorithms in their conventional form, where each integration point is treated separately, leads to severe mesh dependency when a constitutive model with strain-softening is adopted. A non-local definition or a spatial averaging in which the softening rules for each given point are related to the response of its neighbouring points can be introduced to eliminate the effect of mesh size [5]. Here, the quasi-static visco-plastic method was modified using a non-local average with a weight function over $\dot{\varepsilon}^{vp}$ which determines the magnitude of the visco-plastic strain rate.

$$\overline{\dot{\varepsilon}}^{vp} = \int_{\overline{V}} \dot{\varepsilon}^{vp}(x_i)\omega(x_i - x^0_i)dV \qquad \text{with} \qquad \int_{\overline{V}} \omega(x_i - x^0_i)dV = 1 \tag{5}$$

where x_i represents the generic coordinates of any point in the space, x^0_i is the coordinates of the target point with respect to which the spatial averaging is computed, $\dot{\varepsilon}^{vp}(x_i)$ is the local value of $\dot{\varepsilon}^{vp}$, $\overline{V}$ is a circle defined with a center at the target point and radius R, and ω is the weight function, usually the Gaussian distribution, which satisfies the normalizing condition. Fig. 2 shows how GPs or MPs in the mixed integration technique are considered for weighted averaging around the integration point.

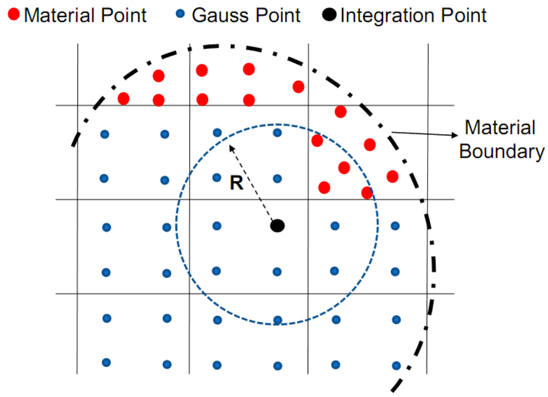

Fig. 2. Description of a continuum using MPM.

In this study, the Mohr-Coulomb failure criterion was implemented and the softening response was accounted for, in a simple though effective way, by gradual linear reduction in strength parameters. The state variable (k_s) which controls the reduction in these parameters can be derived as a function of deviatoric plastic strain as it was proposed by Troncone [6].

$$k_s = \int \dot{k}_s dt \qquad where \qquad \dot{k}_s = \sqrt{0.5\dot{e}^p_{ij}\dot{e}^p_{ij}} \qquad and \qquad \dot{e}^p_{ij} = \dot{\varepsilon}^{vp}_{ij} - \frac{1}{3}\dot{\varepsilon}^{vp}_{kk}\delta_{ij} \tag{6}$$

where $\dot{e}^{vp}_{ij}$ is the rate of deviatoric plastic strain tensor and δ_{ij} is the Kronecker delta. Such a definition for the softening state variable is suitable for large deformation simulation as it is a scalar value and consequently frame independent. The state variable k_s is mapped onto the Gauss points and the strength parameters, cohesion and angle of internal friction, which are functions of k_s, are determined accordingly. It should be noted that the determination of softening parameters and the internal length based on experimental studies are out of the scope of this paper. For more details on the implementation of the Mohr-Coulomb model in visco-plasticity theory readers are referred to Cormeau [11].

4. Slope failure

Slope stability is one of the traditional geotechnical engineering problems which still can be considered as an open topic due to the complexities involved in predicting progressive yielding and slope failure. Here, the proposed MPM formulation was used to simulate progressive failure of a slope to demonstrate the performance of the MPM formulation compared to FEM.

The geometry and boundary conditions of the slope model are presented in Fig. 3. It can be seen that no movement is permitted at the base of the slope, and only vertical movement is allowed at the right lateral boundary. The values of the selected soil parameters are: peak angle of shearing resistance is $\phi'_p = 30^o$, at $k_s = 0\%$ shear strain and a residual angle of shearing resistance $\phi'_p = 24^o$ at $k_s = 15\%$ shear strain, the peak and residual cohesion are $c'_p = c'_r = 1$ kPa, and dilation angle $\psi = 0^o$. The stiffness of the soil is assumed to be elastic with a constant Young's modulus $E = 1.0 \times 10^4$ kPa, Poisson's ratio $\nu = 0.2$ and a bulk density of $\rho = 2000$ kg/m^3. Two slope models consisting of 240 and 540 Q8 elements were generated as coarse and fine meshes, in which each element is filled with 9 material points. The slope is assumed to remain drained and the loading was performed by gradually increasing the gravitational force until failure occurred under a large deformation assumption. In reality such failure can occur due to, for example a gradual increase in the level of the water table. The slope material was modelled using local and non-local plasticity with an internal length (R) equal to 1 m.

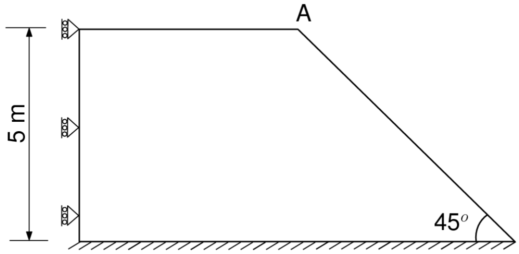

Fig. 3. Geometry and boundary conditions for the slope stability example.

Fig. 4a shows the displacement versus the gravitational acceleration at the surface monitoring point (point *A* on Fig. 3). Considering the vertical displacement at this point for a gravitational acceleration of 6 m/s^2, the local plasticity produced a value of approximately 0.06 m and 0.25 m for the coarse and fine mesh respectively. In a slope stability problem, several elements could undergo failure before a shear band is fully developed. In a finer mesh, when the material is strain-softening with the same applied load more elements fail, which results in faster formation of the slope shear band. This increases the slope deformation at the same load level for finer mesh compared to a coarse mesh. As expected, significant mesh dependency is observed when the local plasticity method is adopted. In contrast, when a non-local technique is applied, there are very small variations between the load-displacement curves computed for the coarse and fine meshes, as can be seen in Fig. 4. This proves that strain softening materials require a non-local approach to account for scale effects and possible mesh size dependency, therefore resulting in a more viable method for quantifying progressive failure in slope stability.

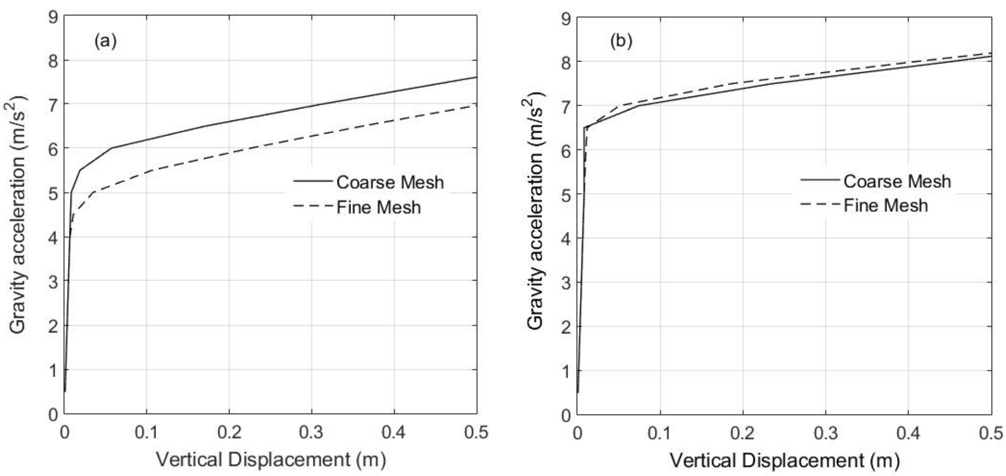

Fig. 4. Vertical displacement at the top of the slope using (a) local and (b) non-local plasticity algorithms.

Fig. 5 shows the development of accumulated k_s at different loading stages for the finer mesh with non-local plasticity. As expected, the rate of softening increased at the toe of the slope in the early stages of the slope loading, with the failing soil mass progressively extending towards the top surface slope as the gravitational force is increased. It may be noted that the MPM local iteration process also exhibited good convergence response and the simulation proceeded to completion even in the presence of excessive deformation in the initial geometry at no extra computational cost.

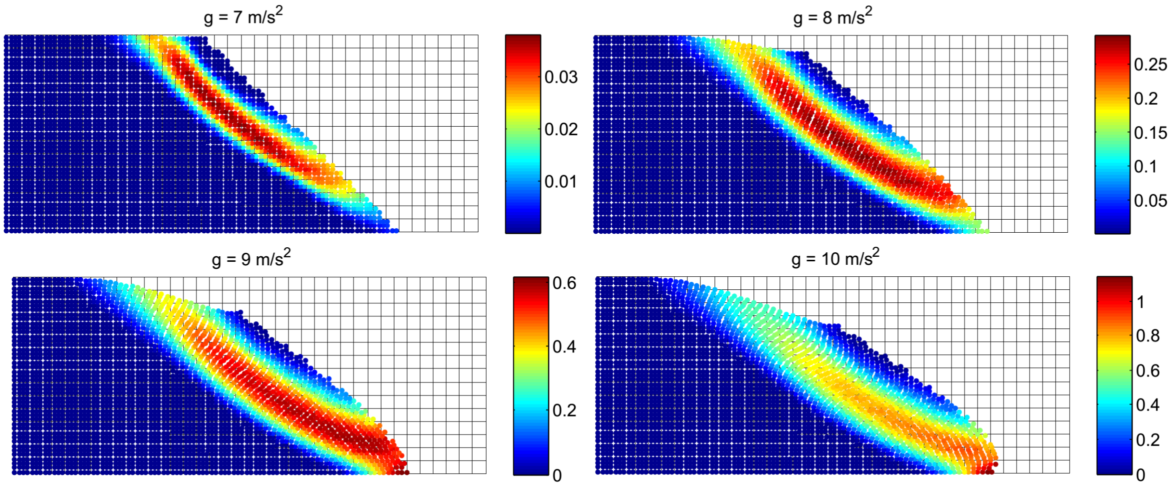

Fig. 5. Accumulated plastic shear strain (k_s) in the slope for different gravitational accelerations.

5. Conclusion

In this paper, an implicit quasi-static MPM equipped with a non-local plasticity algorithm was used for quasi-static geotechnical engineering problems involving with excessive deformation. Such a formulation has several advantages over the conventional explicit time integration MPM. For instance, there is no need for numerous iterations to damp the acceleration for achieving convergence in quasi-static simulations; moreover, higher-order elements can be adopted to generate more accurate results for advanced constitutive models or fluid-coupled simulations. Non-local plasticity was formulated and incorporated into the code to eliminate the severe mesh dependency observed in numerical simulation of strain softening constitutive models using conventional local plasticity. Only one extra constitutive parameter, the internal length, is added which controls the radius of the spatial averaging. The non-local plasticity algorithm provides almost identical results for different mesh sizes when the soil softening response is taken into account. By means of an example of progressive failure of a slope, it has been shown that the non-local plasticity performed very well and produced similar results for both coarse and fine meshes. MPM simulation proceeded to completion even in the presence of excessive deformation in the initial geometry without any mesh distortion problem associated with the updated Langrangian FEM formulation at no extra computational cost.

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