



1st International Conference on the Material Point Method, MPM 2017

# Numerical simulation of deep vibration compaction in Abaqus/CEL and MPM

Alexander Chmelnizkij<sup>a,\*</sup>, Sparsha Nagula<sup>a</sup>, Jürgen Grabe<sup>a</sup>

<sup>a</sup>*Institute of Geotechnical Engineering and Construction Management, Hamburg University of Technology, Hamburg 21079, Germany.*

---

## Abstract

A numerical model is set up to compare CEL and MPM simulating the densification of dry sand using the deep vibration compaction method. This compaction method densifies loose sands by means of shear deformation processes imparted by horizontal vibrations of vibrator probe at the required soil depth. Both methods are capable to simulate large deformation without the drawbacks of mesh distortion. A hypoplastic constitutive model is used to characterize the stress-strain behaviour of the sand. Modelling parameters in both numerical domains were tried to keep similar to the possible extent so as to have comparable results. Both of the numerical methods were compared in light of the void ratio, displacement and stress distribution in the sand layer after being subjected to 20 cycles of vibration. It was observed that both the models were able to model large scale deformations quite satisfactorily and yielded comparable qualitative results. The differences in the outcomes of both approaches were examined for probable reasons of disparity.

© 2016 The Authors. Published by Elsevier Ltd.

Peer-review under responsibility of the organizing committee of the 1 st International Conference on the Material Point Method.

*Keywords:* material point method; CEL; deepvibration compaction; hypoplastic.

---

## 1. Introduction

The deep vibration compaction method is an established ground improvement technique. This technique is used to improve the properties of loose to medium dense granular soils by compacting deep layers of the soil and therefore reducing settlements and increasing the vertical bearing capacity of foundations. A deep vibrator compactor generally consists of a steel tube (the vibrator) with a length of 2.0 to 4.0 m and a diameter of 0.30 to 0.50 m which

---

\* Corresponding author.

*E-mail address:* [alexander.chmelnizkij@tuhh.de](mailto:alexander.chmelnizkij@tuhh.de)

represents the main part of the deep vibration compactor. A heavy mass is located inside the tube which rotates around the vertical axis of the compactor creating vibration forces. This results in an oscillating movement of the compactor during the compaction process. The tube is pin jointed to a stay tube, which has the same diameter as the steel tube and is carried by a crane [2]. A typical compactor has a vibration frequency from 15 to 60 Hz, which allows a maximum deflection of the vibrator at its toe of 0.003 to 0.021 m. This is equivalent to a horizontal force of 150 to 700 kN [1, 2]. The vibrator penetrates to the desired installation depth by its dead load or with the help of a drilling fluid. During the compaction of the surrounding soil the oscillating vibrator is pulled up stepwise. Within each step with a duration of 30 to 60 s the vibrator can be pulled up between 0.3 to 1.0 m. Shear waves in the soil resulting from the energy of the vibrator lead to the compaction of the soil. The soil from adjacent regions will move into the resulting void spaces. This movement can be noticed as a settlement of the ground surface. To compensate the settlement the resulting funnel is refilled with material. This technique leads to a dense packing of the soil in an area with a radius of 0.6 until 1.75 m around the compactor [1, 2]. Numerical simulations of the method would help to develop an insight and also to optimize the technique. Numerical simulations of the deep vibration compaction technique using the Coupled Eulerian Lagrangian CEL approach has been carried out. This work aims to compare the CEL and Material Point Method (MPM) approach for simulating the deep vibration compaction in loose sand. Therefore an undrained simulation with hypoplastic material behavior of the sand was performed. As both methods differs in many points e.g. element type, integration schemes, meshing algorithm, pre- and post-processing etc. the goal is first to compare the results qualitatively and then interpret the differences.

## 2. Numerical modeling

### 2.1. Coupled Eulerian-Lagrangian (CEL)

The simulation of the deep vibration compaction method is a problem involving large deformations. The coupled Eulerian-Lagrangian (CEL) method is a numerical technique which can be used for this type of problem. This method combines the advantages of the Lagrangian analysis with those of an Eulerian formulation. A characteristic of the Lagrangian formulation is the deformable mesh which moves with the material meaning that the movement of a continuum is described as a function of time and material coordinates. In an Eulerian analysis on the other hand the movement of a continuum is formulated by a function of time and spatial coordinates. The mesh of an Eulerian formulation remains undeformed and the material can move freely through the mesh. Both techniques are depicted in Fig. 1 [3].

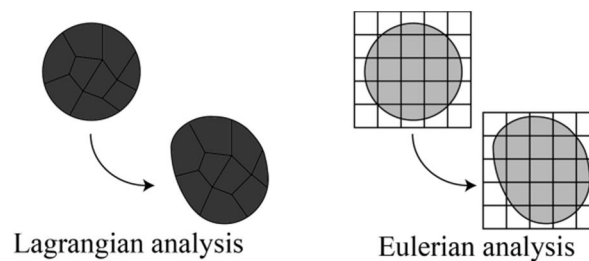


Fig. 1. Deformation of a continuum in a Lagrangian (left) and an Eulerian analysis (right) [3].

During a coupled Eulerian-Lagrangian analysis a Lagrangian object can move inside an Eulerian region. The material distribution inside the Eulerian region is defined by the Eulerian Volume Fraction (EVF). An element can take all states between completely filled with material ( $EVF = 1$ ) or being empty ( $EVF = 0$ ). The Lagrangian object can move through the region without resistance until it touches Eulerian material. Then the contact algorithm starts to act. The algorithm is implemented as a general contact formulation based on the penalty method and therefore, assumes a hard pressure-overclosure behavior, is less strict than a kinematic method and allows small penetrations of the Eulerian material into the Lagrangian object (compare Fig. 2).

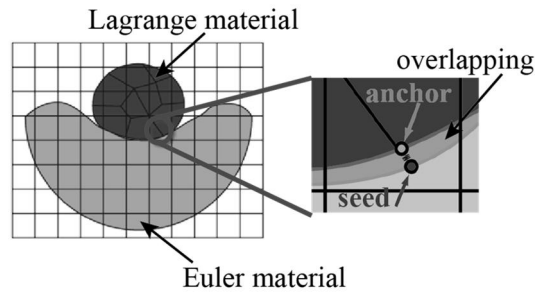


Fig. 2. Illustration of the penalty contact method [3].

For each time increment the Lagrangian phase is calculated first. Followed by the Eulerian phase where the movement and the position of the Eulerian material are determined. To avoid element distortion during the Lagrangian phase the time step has to be sufficiently small. The integration scheme for the calculation is an explicit one, meaning that the solution only depends on the result of the previous time step.

## 2.2. CEL model

A three-dimensional model based on the CEL method with 57540 hexahedral elements as depicted in Figure 3 is created. The soil body has a radius of 30.0 m and a height of 30.0 m. The bottom boundary of the soil domain is fixed and the vertical boundaries are free to move vertically and lateral movements are restrained. A void area of 1.0 m height is located above the soil to allow the material to move inside this space during the simulation. The vibrator is modeled 10.0 m preinstalled into the soil by using the wished-in-place technique and has a length of 4 m and a diameter of 0.50 m. The vibrator is vertically hinged to the stay tube which has the same diameter as the vibrator. The stay tube is fixed in horizontal and vertical direction at its upper end. Both tools are modeled with a linear elastic material behavior and the material parameters of steel. The contact between the compactor and the soil is frictionless assuming the influence of friction between the moving soil and the compactor is negligible. The movement of the vibrator is implemented as a sinusoidal, horizontal force with an amplitude of 700 kN and a frequency of 30 Hz at the tip of the probe which is prescribed to the central node at the tip of the vibrator.

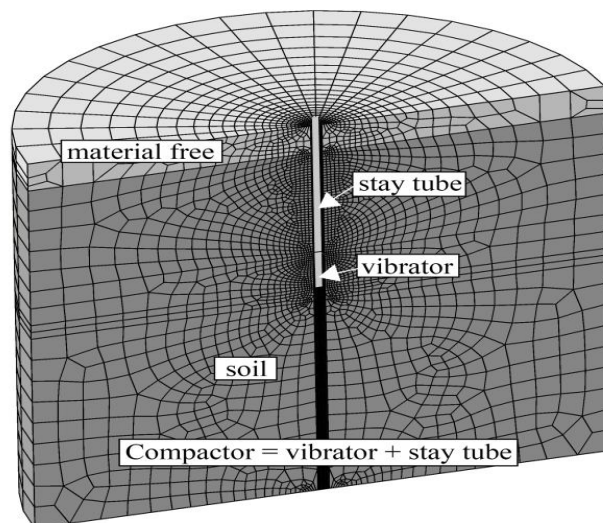


Fig. 3. Vertical cut through the Finite Element model to simulate the deep vibration compaction.

### 2.3. Material point method (MPM)

The Material Point Method (MPM) is based on the Finite Element Method (FEM) and was originally developed as the Particle-In-Cell method for fluid dynamics [4]. During one time step first a Lagrangian step is performed by deforming the mesh together with the material points. Afterwards the mesh is reset to its initial configuration, while the material points remain their position. MPM allows to simulate large deformations in a similar way like CEL, by letting the material points flow through an unchanged background mesh as shown in Figure 4. The unknown field variables e.g. acceleration and velocity are calculated in the nodes of the background mesh and evaluated by the shape functions inside an element at the positions of the material points. The material properties are stored in the material points and updated every time step by a constitutive model. MPM is recently more and more applied to geotechnical problems [5].

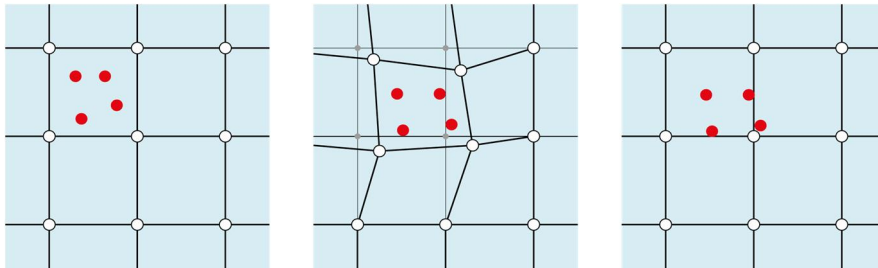


Fig. 4. Distribution of material points in an arbitrary element (left), Lagrangian step (center), Convective step by resetting the mesh (right).

One of the first applications can be found in [6]. For this paper the joint MPM code of the MPM Research Community is used, which is being developed by the University of Cambridge, UPC Barcelona, Hamburg University of Technology, University of Padova, Delft University of Technology and Deltares.

### 2.4. MPM model

The geometry for the MPM calculation is shown in Figure 5. Similar to CEL a region of void elements needs to be defined allowing the material points to move freely at the soil surface. The dimensions and boundary conditions were taken from section 2.2.

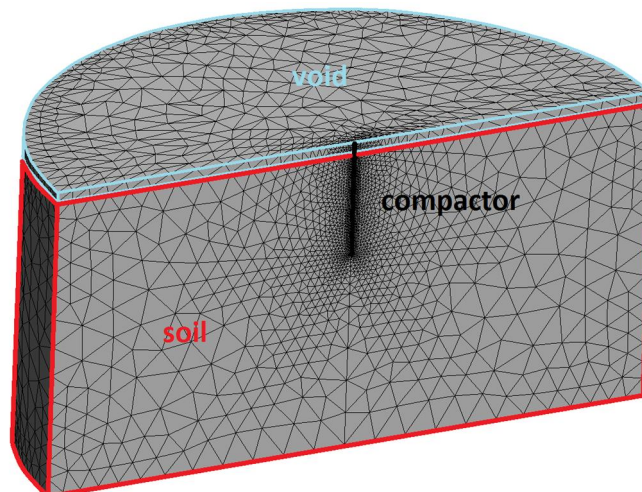


Fig. 5. Geometry of the computational mesh for the MPM simulation.

The dimensions and boundary conditions were taken from section 2.2. The computational domain was discretized by a non-uniform mesh to ensure high accuracy close the compactor and still being efficient by increasing the element size in vicinity of the boundaries. The mesh in Figure 5 consists of 54050 tetrahedral elements. Initially each element has four material points representing the material e.g. soil and steel.

### 2.5. Hypoplastic constitutive model

To model the soil behavior realistically a high class constitutive model is required, in both numerical models the hypoplastic model was used. For granular soils the hypoplastic constitutive model according to von Wolffersdorff [7] with the extension for intergranular strain by Niemuns and Herle [8] is suitable. With the help of this model it is possible to describe the nonlinear and inelastic behavior of soil. This includes dilatancy, contractancy, the dependency of stiffness and strength on the pressure and the void ratio, as well as different stiffnesses for loading, unloading and reloading. The utilized material parameters for Mai-Liao-Sand with a relative density of  $I_D = 0.2$  can be found in Henke et al. (2012).

Table 1: Hypoplastic model parameters for Mai-Liao sand.

| Material Parameter | Value  | Description                                 |
|--------------------|--------|---|
| $\varphi_c$        | 31.5   | Critical state friction angle [°]           |
| $h_s$              | 32     | Granular hardness [MPa]                     |
| $n$                | 0.324  | Exponent                                    |
| $e_{d0}$           | 0.57   | Minimum void ratio                          |
| $e_{c0}$           | 1.04   | Critical void ratio                         |
| $e_{i0}$           | 1.20   | Maximum void ratio                          |
| $\alpha$           | 0.4    | Exponent                                    |
| $\beta$            | 1.0    | Exponent                                    |
| $m_T$              | 2.0    | Stiffness ratio at 90° change of direction  |
| $m_R$              | 5.0    | Stiffness ratio at 180° change of direction |
| $R$                | 0.0001 | Maximum value of intergranular strain       |
| $\beta_R$          | 0.5    | Exponent                                    |
| $X$                | 6.0    | Exponent                                    |

## 3. Comparison of numerical results

### 3.1. Void ratio

The void ratio in both models was set to a value of 0.93 lesser than the critical void ratio at the beginning of the vibration. This corresponds to a loose sand. The variation in the void ratio after the 20 cycles of vibrations as obtained from both models is qualitatively compared in Figure 6. The void ratio gradually reduces as the distance from the probe increases forming a bulb. This bulb represents the radius of influence of the vibrating probe, beyond which the vibrations of the probe hardly lead to any densification. It can be observed from the Figure 6 that both models lead to development of a bulb around the probe though the form of the densification bulb in the MPM results is not as sharp as in the CEL results. This can be attributed to the fact that the continuum based approach of the MPM model needs further refinement to model the gaps between soil and vibrator during vibrations.

Figure 7 compares the horizontal distribution of the void ratio at the tip of the probe in a depth of 9 meters. It can be observed in Figure 7 that in CEL the void ratio near the probe rises above the native value of 0.93 and gradually reduces. Beyond the radius of influence which is nearly around 1.6 m in both the cases, the effect eases.

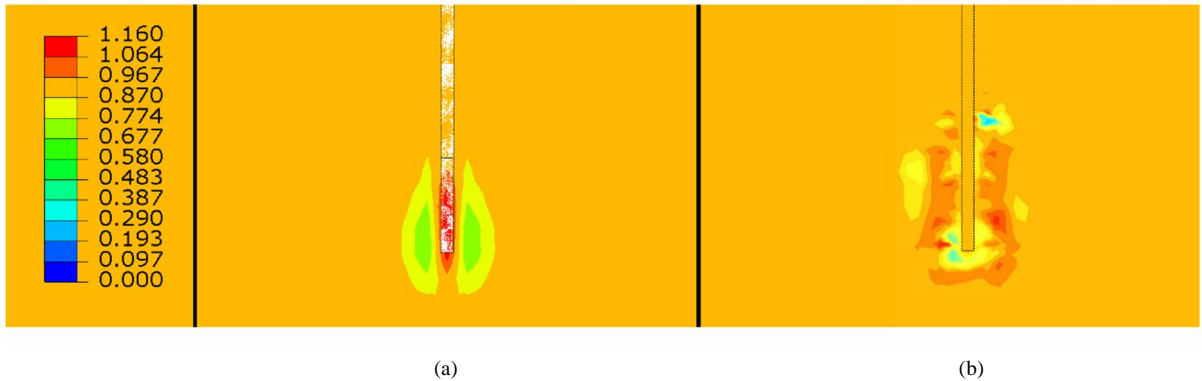


Fig. 6. Void ratio distribution after 20 load cycles (a) CEL model (b) MPM model.

An interesting difference is that in the CEL simulation the soil tend to loosen close to the vibrator while in the MPM simulation it's compacting. The influence radius of the vibrator is similar in both simulations. It is also worth to mention that the results of the MPM simulation are oscillating while the CEL method produces very smooth curves. The main reason for this behavior is the artificial damping in particular the bulk viscosity which was used in the CEL simulation. This feature is not available in the used MPM code.

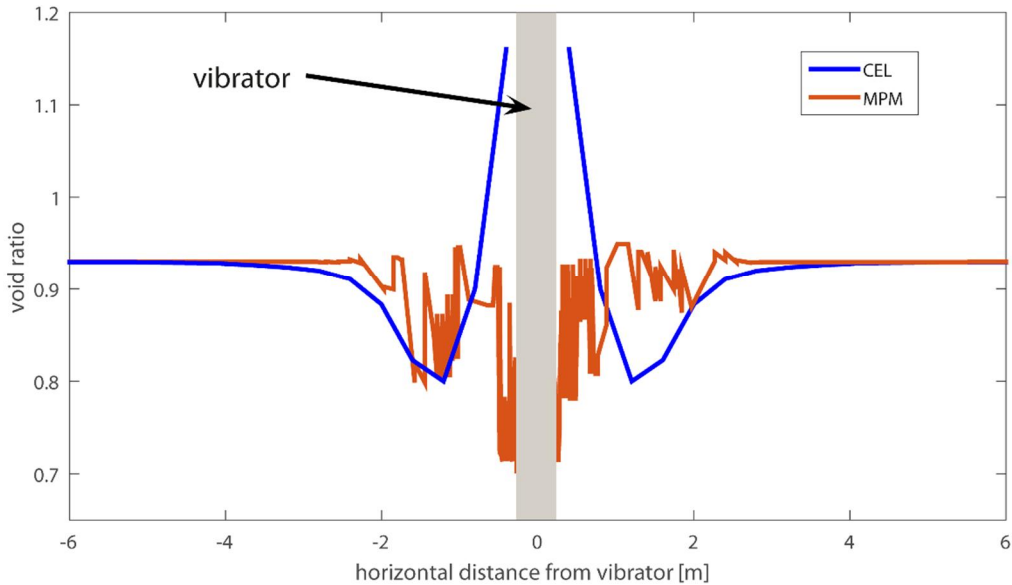


Fig. 7. Void ratio of both simulations after 20 cycles in 10m depth.

### 3.2. Vertical displacement of sand

Shear waves in the soil resulting from the energy of the vibrator lead to the compaction of the soil. The soil from adjacent regions will move into the resulting void spaces. This movement can be noticed as a settlement of the ground surface. Figure 8 indicates that both models predict the movement of the upper sand downward and towards the probe leading to the filling of the void spaces created by the compaction of the loose sand around the probe. The maximal amplitude of the vertical displacements in the MPM simulation is 3 times higher than in the CEL simulation. Both methods also form an upward moving part directly under the vibrator probe.



Fig. 8. Vertical displacement of sand after 20 load cycles (a) CEL model (b) MPM model.

### 3.3. Horizontal stresses

The vibrations lead to the reduction of horizontal stress in the sand around the probe. It can be observed in Figure 9 that in both models the horizontal stress contour around the probe forms a bulb similar to the compaction zone, having a magnitude of nearly zero indicating the effect of the shear waves created due to the vibration of the probe.

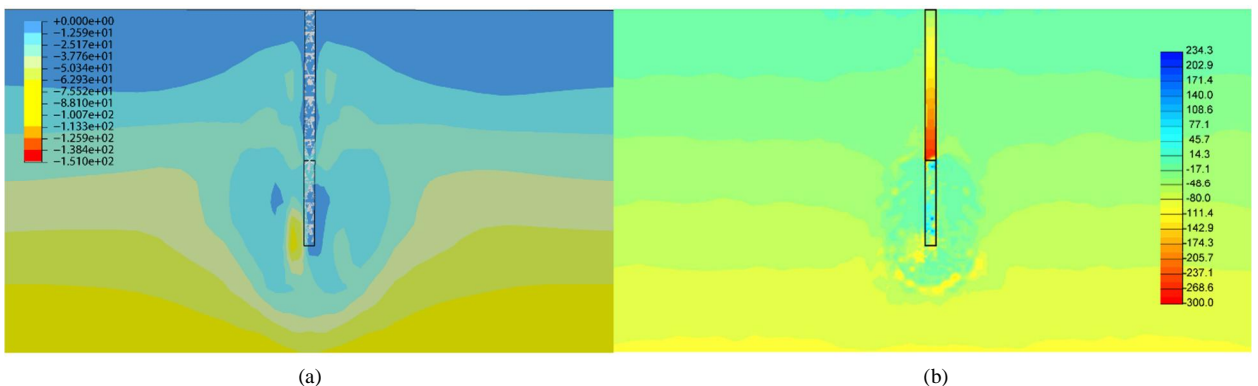


Fig. 9. Horizontal stress [kN/m<sup>2</sup>] after 20 load cycles (a) CEL model (b) MPM model.

Due to the combination of Eulerian and Lagrangian approach in CEL only the soil stresses are visualized in Figure 9(a) whereas in MPM only Lagrangian stresses are present and so soil and pile stresses are shown in Figure 9(b). The probe is subjected to tensile stresses due to the prescribed forces. The horizontal stresses near the probe tend to zero in both simulations.

#### 4. Conclusions

It could be observed that both methods are suitable for the simulation of the mechanical processes due to deep vibration compaction. The physical behavior of the sand and vibrator were successfully simulated in both cases. Nevertheless the computed variables differ from each other in distribution and magnitude. The biggest difference could be observed in the comparison of vertical displacements. In both methods the displacements are the results of numerical integration schemes. In the case of CEL it's a combination of central and forward differences in the case of MPM it's the semi-explicit Euler-Cromer-Scheme. In contrast to the displacements the horizontal stresses in the sand were almost the same in both cases. For further comparisons both models need to match more for example by implementing the same integration scheme or element type or perform the simulation on an identical mesh.

#### References

- [1] Fellin, W. (2000) *Rütteldruckverdichtung als plastodynamisches Problem*. Institute of Geotechnics and Tunneling, University of Innsbruck, Advances in Geotechnical Engineering and Tunneling, Heft 2.
- [2] Witt, K.J. (2009) *Grundbau-Taschenbuch, Teil 2: Geotechnische Verfahren*, Ernst&Sohn, Berlin, 2009.
- [3] Qiu, G. (2012) *Coupled Eulerian Lagrangian Simulations of Selected Soil-Structure Interaction Problems*. Veröffentlichungen des Instituts für Geotechnik und Baubetrieb der TU Hamburg-Harburg, Heft 24.
- [4] Harlow, F. H. (1964). "The particle-in-cell computing method for fluid dynamics". *Methods Comput Phys*; 3(3):319–43.
- [5] Yerro A., Alonso E.E., Pinyol N. (2013). The Material Point Method: A promising computational tool in Geotechnics. In: Proc.18th International Conference for Soil Mechanics and Geotechnical Engineering, Paris. Vol. 1, pp. 853-856.
- [6] Więckowski, Z., Youn, S. K. & Yeon, J. H. (1999). "A particle-in-cell solution to the silo discharging problem". *Int J NumerMeth Eng*; 45(9):1203–25.
- [7] Von Wolffersdorff, P. (1996) A hypoplastic relation for granular materials with a predefined limit state surface. *Mechanics of Cohesive Frictional Materials*, Vol:1, pp 279-299.
- [8] Niemunis, A. and Herle, I. (1997) Hypoplastic model for cohesionless soils with elastic strain range. *Mechanics of Cohesive-Frictional Materials*, Vol:2, pp 279-299
- [9] Henke, S. and Hamann, T. and Grabe, J. (2012) Numerische Untersuchungen zur Bodenverdichtung mittels Rütteldruckverfahren, *Proc. of 2. Symposiums Baugrundverbesserung in der Geotechnik 2012*, Wien, pp 209-228.